Reflection and Transmission Coefficient

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Abstract

Searching for "reflection coefficient" yields immediate results. However, the transmission coefficient is often never mentioned. In any practical problem involving scattering of waves in non-uniform structures, it is important to be able to calculate both the forward and reverse waves at each scattering site. Although many engineers take the reflection coefficient as a given, it is instructive to note that the expression is uniquely determined by just two principles. These are 1) voltage continuity at the scattering boundary, and 2) conservation of power between the incident wave and the two scattered waves.

1 Scattering at a step change in impedance

Consider a wavefront of amplitude 1 propagating left to right in a transmission with impedance Z_A, impinging on a step in impedance to Z_B(figure 1).

Two conditions are sufficient for calculation the reflection coefficient (ratio of reflected voltage to incident voltage), and transmission coefficient (transmitted voltage divided by incident voltage). The first is voltage continuity at the scattering boundary. The second is power conservation: the sum of the power of the reflected and transmitted waves must equal the power of the incident wave.

1.1 Voltage continuity

The voltage at any point in a transmission line is the sum of the forward wave plus the reflected wave at that point. The voltage at both sides of the discontinuity must match. This implies that the sum of the incident plus reflected amplitudes must equal the transmitted amplitude at the discontinuity. If we postulate a reflected fraction of the incident signal α, then the transmitted signal amplitude must be $1 + \alpha$.

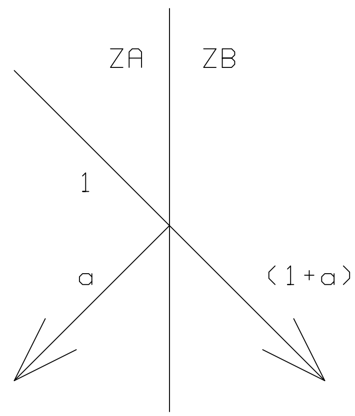

Figure 1: An incident voltage step propagating in a medium with impedance Za impinges on a step transition to impedance Zb, resulting in a reflected wave with amplitude a and a transmitted wave of amplitude (1+a).

1.2 Power conservation

The second condition is that the power in the incident signal must equal the sum of power in the reflected and transmitted signal.

$$P_{incident} = P_{reflected} + P_{transmitted}$$

$$\frac{1}{Z_A} = \frac{\alpha^2}{Z_A} + \frac{(1+\alpha)^2}{Z_B}$$

Solving the quadratic for α yields the reflection coefficient:

$$\alpha = \frac{Z_B - Z_A}{Z_B + Z_A}$$

The transmission coefficient is then

$$1 + \alpha = \frac{2Z_B}{Z_B + Z_A}$$